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CONSTRUCTION OF GENERALIZED DIRECTED ASSOCIATION SCHEME FROM COMPLETE BIPARTITE GRAPH

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Abstract

In this paper methods of construction of some class of Generalized Directed Association Scheme from complete bipartite graph is giventhen an approach to construct class Generalised Directed Association Scheme from complete n-partite graph is suggested .

Keywords:

Association Scheme;
Bipartite graph.

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1. Introduction :

1.1 Association Scheme: An Association Scheme on a finite set X is define as a patition $C = \{C_1, C_2, C_3, \dots, C_n\}$ of $X \times X$ which satisfies the following properties:

(i) $C_0 = \{(x,x): x \in X\}$

(ii) For each $i \in \{1,2,3,\dots,n\}$ $C_i = C_i^{-1}$ where $C_i^{-1} = \{(y,x): (x,y) \in C_i\}$.

(iii) There exist a non negative integer p_{ij}^k for $0 \le i,j \le n$ such that for $(x,z) \in C_k$, the number of elements in the set $S = \{y: (x,y) \in C_i \text{ and } (y,z) \in C_j\}$

(iv) is equal to p_{ij}^k and this value is independent of the choice of $(x,z) \in C_k$.

(v) (Vide [1] and [2])

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1.2 Generalized Directed Association Scheme (GDAS): In 2011 Singh and Manjhi defined a generalization of association scheme known as Generalized Directed Association Scheme (GDAS) as a collection $C = \{C_1, C_2, C_3, ..., C_m\}$ of subsets of $X \times X$ on a finite set X which satisfies the following properties:

(i) $\bigcup_{i=1}^{n} C_i = X \times X$

(ii) There exist a non negative integer p_{ij}^{k} $for\ 0 \leq i, j \leq m$ such that for (x,z)∈ C_k the number of elements in the set $S = \{y: (x, y) \in C_i \text{ and } (y, z) \in C_j\}$ is equal to p_{ij}^{k} and it is independent of the choice of (x,z)∈ C_k

Another definition of GDAS in terms of adjacency matrices $M_1, M_2, M_3, \dots, M_n$ of $C_1, C_2, C_3, \dots, C_n$ respectively took the following form:

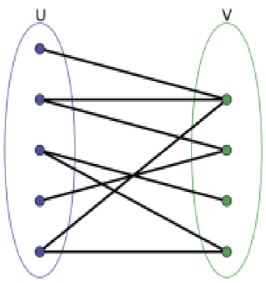
(i) $\sum_{i=1}^{n} M_i = J = all\ one\ matrix\ of\ order\ |X|$, where X is the finite set over which GDAS is defined.

(ii) $M_i M_j = \sum_{i=1}^{m} P_{ij}^{k}$ where p_{ij}^{k} are non - negative integers.

(Vide [3])

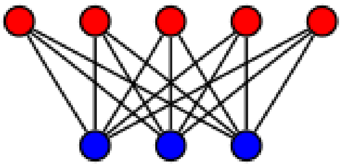
1.2 Bipartite graph: A bipartite graph is a simple graph in which vertices can be divided into two parts so that every edge connects a vertex of one part to a vertex of another part.

Example:



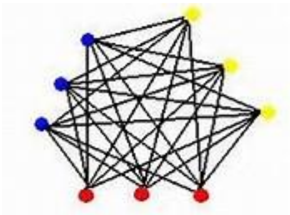
1.3 Complete bipartite graph: A complete bipartite graph is a bipartite graph in which each vertex of one part is connected by an edge to each vertex of another part. The division of set of vertices is called bipartition. If a complete bipartite graph has bipartition (X, Y) with $|X| = r$ and $|Y| = s$ then it is denoted by $K_{r,s}$.

For example: $K_{5,3}$ is



1.4 Complete n-partite graph: A complete n-partite graph is simple graph in which set of vertices can be divided into n-parts $X_1, X_2, X_3,..., X_n$ so that each vertex of X_i is connected by an edge to each vertex of X_j *for* $i \neq j$ and $i, j \in \{1,2,3,..., n\}$.

Example of a tripartite graph $K_{3,3,3}$ is



Reference for 1.3, 1.4 and 1.5 is [4]

2. MAIN WORK:

In this paper I forward methods of construction of a class of Generalized Directed Association Scheme (GDAS) from complete bipartite graph $K_{n,m}$.

2.1 construction of a class of Generalized Directed Association Scheme from complete bipartite graph $K_{n,m}$

Consider $K_{n,m}$ where $X_1 = \{u_1, u_2,..., u_n\}$ and $X_2 = \{v_1, v_2,..., v_m\}$ are bipartition of the set of vertices $X = \{u_1, u_2,..., u_n, v_1, v_2,..., v_m\}$ and construct the following four sets:

$$C_1 = \{(x, y) : if\ x \in X_1 \text{ and } y \in X_2\}$$
$$C_2 = \{(x, y) : if\ y \in X_1 \text{ and } x \in X_2\}$$
$$C_3 = \{(x, y) : if\ x, y \in X_1\}$$
$$C_4 = \{(x, y) : if\ x, y \in X_2\}$$

Let M_1, M_2, M_3 and M_4 be adjacency matrices of C_1, C_2, C_3 and C_4 respectively then

$M_1 = \begin{bmatrix} 0 & J_{nm} \\ 0 & 0 \end{bmatrix}$, $M_2 = \begin{bmatrix} 0 & 0 \\ J_{mn} & 0 \end{bmatrix}$, $M_3 = \begin{bmatrix} J_{nn} & 0 \\ 0 & 0 \end{bmatrix}$ and $M_3 = \begin{bmatrix} 0 & 0 \\ 0 & J_{mm} \end{bmatrix}$ where J_{uv} is a all 1 matrix of order $u \times v$ $\forall u, v \in \{n, m\}$ and 0 are the zero matrices of suitable size so that each $M_i\ (i = 1,2,3,4)$ is a square matrix of order $(m + n)$.

We see the following calculations:

$1. M_1M_2 = mM_3, M_2M_1 = nM_4$

$2. M_1M_3 = 0, M_3M_1 = nM_1$

$3. M_1M_4 = mM_1, M_4M_1 = 0$

$4. M_2M_3 = nM_2, M_3M_2 = 0$

$5. M_2M_4 = 0, M_4M_2 = 3M_2$

$6. M_3M_4 = 0 = M_4M_3$

$7. M_i^2 = 0 \; for\, i = 1,2$

$8. M_3^2 = 2M_3, M_4^2 = 3M_4$

Here we see that each product $M_iM_j (i, j \in \{1,2,3,4\})$ is a linear combination of M_1, M_2, M_3 and M_4

Therefore $C = \{C_1, C_2, C_3, ..., C_m\}$ is a GDAS.

2.2 construction of a class of Generalized Directed Association Scheme from complete bipartite graph $K_{n,n}$

Consider $K_{n,n}$ where $X_1 = \{u_1, u_2, ..., u_n\}$ and $\mathrm{X}_2 = \{v_1, v_2, ..., v_n\}$ are bipartition of the set of vertices $X = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n\}$ and construct the following two sets:

$C_1 = \{(x, y) : x \in X_1 \; and \; y \in X_2 \; or \; y \in X_1 \; and \; x \in X_2\}$

$C_2 = X \times X - C_1$

Let M_1 and M_2 be adjacency matrices of C_1 and C_2 respectively then

$M_1 = \begin{bmatrix} 0 & J_n \\ J_n & 0 \end{bmatrix}$ and $M_2 = \begin{bmatrix} J_m & 0 \\ 0 & J_m \end{bmatrix}$ where J_u is a squre matrix with each entry 1 $\forall u \in \{n, m\}$ and 0 are the zero matrices of suitable size so that each $M_i \, (i = 1,2,3,4)$ is a square matrix of order $(2n)$.

We see the following calculations:

$1. M_1M_2 = M_2M_1 = M_2$

$2. M_i^2 = M_i \; for\, i = 1,2$

Here we see that each product $M_iM_j (i, j \in \{1,2\})$ is a linear combination of M_1 and M_2

Therefore $C = \{C_1, C_2\}$ is a GDAS.

2.1 construction of a class of Generalized Directed Association Scheme from complete n-partite graph $K\underbrace{r_1, r_2, ..., r_n}_{n\, terms}$

Consider $K\underbrace{r_1, r_2, ..., r_n}_{n\,terms}$ where $X_i = \{u_1^{\ i}, u_2^{\ i}, ..., u_{r_i}^{\ i}\}(i = 1,2,3..., n)$ are n-partition of the set of vertices $X = \bigcup_{i=1}^{n} X_i$ and construct the following n^2 sets:

$$C_{ij} = \{(x, y) : x \in X_i \text{ and } y \in X_j\} \; for\, i \neq j$$
$$C_{ii} = \{(x, y) : x, y \in X_i\} \; \forall i = 1,2,3,4..., n.$$

By the above method we can show that these n^2 sets form a GDAS over the set of vertices X.

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